ADC Depending Limitations for Six-Port Based Distance Measurement Systems

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***Abstract*— This publication will show some considerations for selecting an appropriate analog-digital-converter (ADC) in Six-Port based radar systems for distance measurements. It will be discussed how the ADC resolution is limiting the overall system range resolution. Furthermore, configurations with non-synchronous sampling of the four baseband signals and their influences on the system performance will be investigated.**

***Index Terms*— ADC, distance measurement, low-cost, radar, six-port, synchronous sampling**

I. Introduction

The Six-Port concept was originally introduced by G.F. Engen as an alternative method for vector network analysis [1]. Nowadays it is used for a lot of measuring purposes, such as frequency discrimination [2] as well as distance and vibration measurements in industrial environments [3] and in the medical sector for applications like heartbeat monitoring [4].

The Six-Port can be used in these applications due to its capability of measuring phase differences with high accuracy and resolution. This results from its interferometric behavior in combination with a linear system based on a passive structure and linear power detectors.

However, the phase resolution is limited by, e.g., noise, the radio frequency (RF) power levels, and the analog baseband processing. Within this publication the baseband signal processing will be investigated regarding the selection of a proper ADC. This is especially important for the development of systems in the often addressed low-cost segment, because due to the passive and low complex RF front end the ADC may be one of the most expensive parts.

II. Six-Port based distance measurement

A Six-Port receiver can be understood as a homodyne additive mixer with two RF input ports and four output ports. Within the structure the two input signals will be superimposed with different discrete phase shifts and fed to the outputs. The power at the four output ports is measured by power detectors, e.g., diode detectors, and from these power signals the complex baseband signal can be computed.

Assuming a sinusoidal continuous wave (CW) signal with the same frequency at both input ports and that the detectors are used in their square-law region, this will lead to the following four baseband voltages, depending on the RF signal powers P_1, P_2 at the two input ports and the phase shift φ in between them.

$$B_3 = \frac{1}{4}\left(P_1 + P_2 + 2\sqrt{P_1 P_2}\sin(\varphi)\right) \quad (1)$$

$$B_4 = \frac{1}{4}\left(P_1 + P_2 - 2\sqrt{P_1 P_2}\sin(\varphi)\right) \quad (2)$$

$$B_5 = \frac{1}{4}\left(P_1 + P_2 + 2\sqrt{P_1 P_2}\cos(\varphi)\right) \quad (3)$$

$$B_6 = \frac{1}{4}\left(P_1 + P_2 - 2\sqrt{P_1 P_2}\cos(\varphi)\right) \quad (4)$$

From these voltages a complex vector $\underline{z}$ can be calculated, whose argument corresponds the phase difference between the two RF input signals:

$$\underline{z} = (B_5 - B_6) + \mathrm{j}(B_3 - B_4) \quad (5)$$

$$\arg\{\underline{z}\} = \sigma = \arctan\left(\frac{\sin(\varphi)}{\cos(\varphi)}\right) = \varphi \quad (6)$$

For the case of distance measurements, a setup shown in Fig. 1, the distance d to the target can now be calculated with knowledge of the wavelength λ of the propagating signal and the measured phase angle σ to

$$d = \frac{\sigma}{2\pi} \cdot \frac{\lambda}{2}. \quad (7)$$

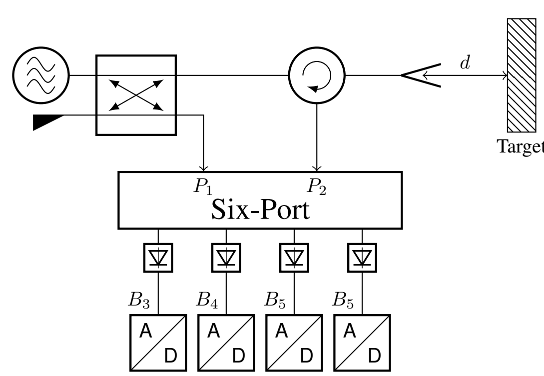

Fig. 1. Simplified concept of Six-Port based radar for distance measurement.

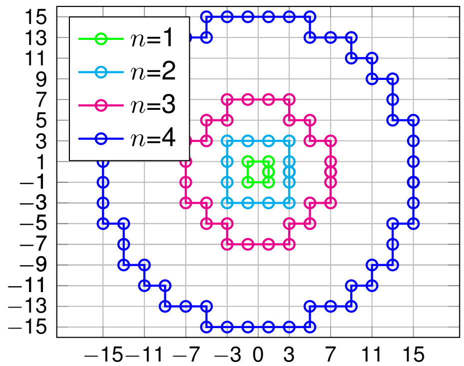

Fig. 2. Discrete complex results in the constellation diagram for full scale input at different numbers of bits.

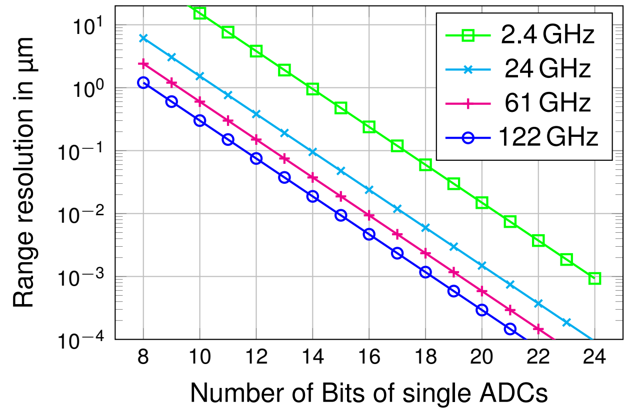

Fig. 3. Maximum reachable resolution over effective number of bits for different system frequencies

III. Discretization

From (7) it is obvious that the achievable distance resolution depends on the discretization of the analog voltage signals from which the phase angle σ is calculated. More precisely, the quantity of discrete steps within the unambiguity range of 2π is limiting the theoretically achievable distance resolution. This number of steps can directly be calculated with knowledge of the resolution of the used ADC. Assuming four identical ADCs digitizing the voltages B_3 to B_6, each of them featuring n Bit, the number of steps N can be calculated to

$$N = 2^{(n+2)} - 3. \tag{8}$$

Fig. 2 shows this relationship in the constellation diagram. For different types of ADCs from 1 to 4 Bit, the discretized results for a phase sweep of σ from 0 to 2π is plotted. From the plot and due to the arctan function it is obvious that the single step heights are not constant and therefore the phase resolution is not equal over the measurement range. Nevertheless, the resulting mean error of the phase measurement due to the discretization within one period can be estimated to

$$\overline{\sigma}_{err} = \frac{2\pi}{N}. \tag{9}$$

For distance measurement setups, a mean resulting distance resolution can be calculated using (7) to

$$\overline{d}_{err} = \frac{\lambda}{2N} = \frac{c}{2fN}. \tag{10}$$

In Fig. 3, this equation is evaluated for several common radar frequencies assuming free space propagation ($c = c_0$), full scale usage of the ADCs and no noise effects. The graph represents the theoretical limit in range resolution over different effective number of bits.

IV. Non-synchronous sampling

Up to now the investigation assumes four parallel, synchronous ADCs, but especially in low-cost system it is interesting to use only a single, or two synchronous ADCs and switch the channels to digitize the signals B_3 to B_6. Especially in static environments or monitoring only slow moving target, there is theoretical no difference between synchronous and sequential digitizing of the single signals.

The situation changes, if the target is moving, because then there is always an error in the detected phase, which depends on the ADC configuration and the velocity of the moving object, respectively.

This behavior can be modeled by an additional phase shift δ, which corresponds to the change in phase between two sampling points. For linear movements between two sample points, what is a good approximation for slow targets or high sample rates, this value depends on the target velocity v and the time t_s between two samples:

$$\delta = t_s v \frac{4\pi}{\lambda} \tag{11}$$

The time t_s is limited by the switching time between two channels or by the sample rate f_s ($t_s = \frac{1}{f_s}$), whatever is lower. Using two synchronous ADC to sample signal B_3 together with B_5 and in a second step B_4 with B_6 yield to the following signals in baseband:

$$B_3' = \frac{1}{4}\left(P_1 + P_2 + 2\sqrt{P_1 P_2}\sin(\varphi)\right) \tag{12}$$

$$B_4' = \frac{1}{4}\left(P_1 + P_2 - 2\sqrt{P_1 P_2}\sin(\varphi + \delta)\right) \tag{13}$$

$$B_5' = \frac{1}{4}\left(P_1 + P_2 + 2\sqrt{P_1 P_2}\cos(\varphi)\right) \tag{14}$$

$$B_6' = \frac{1}{4}\left(P_1 + P_2 - 2\sqrt{P_1 P_2}\cos(\varphi + \delta)\right) \tag{15}$$

In this case, the phase σ' and the resulting error σ'_{err} can be calculated to:

$$\sigma' = \arctan\left(\tan\left(\frac{\delta}{2} + \varphi\right)\right) = \frac{\delta}{2} + \varphi \tag{16}$$

$$\sigma'_{err} = \sigma' - \varphi = \frac{\delta}{2} \tag{17}$$

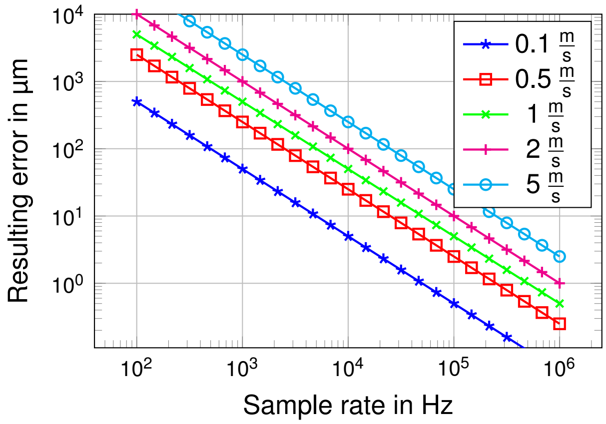

Fig. 4. Worst case error for the best dual synchronous ADC configuration.

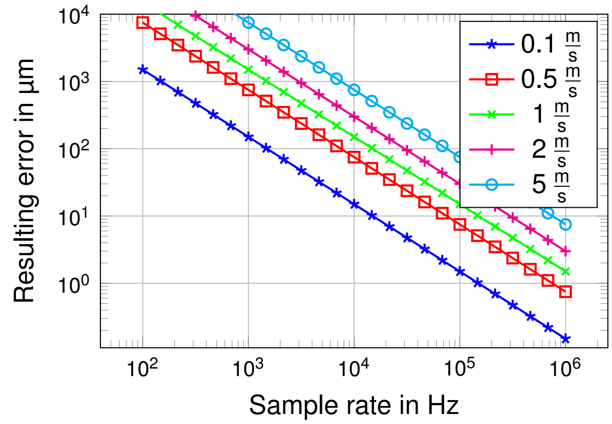

Fig. 5. Worst case error for the best single ADC configuration.

Thus, for this configuration with two synchronous ADCs a constant error of $\frac{\delta}{2}$ can be observed. The other port combinations for the sampling sequences have also been investigated, but lead to worse results.

Fig. 4 shows the resulting error in distance introduced by the sequential sampling for the shown dual-ADC configuration. Combining (7), (17) and (11) it is obvious, that this evaluation does not depend on the used transmission frequency. The resulting error can be neglected, if the target is nearly static or the sampling frequency is high enough. The simulation stops at a sample rate of 1 MSample/s due to the primary usage of such systems in low-cost applications.

In the case of using only a single ADC to convert the four signals, there are a lot of possible sequences. The best found sampling sequence is B_3, B_5, B_6, B_4 producing the following baseband signals:

$$B_3'' = \frac{1}{4}\left(P_1 + P_2 + 2\sqrt{P_1 P_2}\sin(\varphi)\right) \tag{18}$$

$$B_4'' = \frac{1}{4}\left(P_1 + P_2 - 2\sqrt{P_1 P_2}\sin(\varphi + 3\delta)\right) \tag{19}$$

$$B_5'' = \frac{1}{4}\left(P_1 + P_2 + 2\sqrt{P_1 P_2}\cos(\varphi + \delta)\right) \tag{20}$$

$$B_6'' = \frac{1}{4}\left(P_1 + P_2 - 2\sqrt{P_1 P_2}\cos(\varphi + 2\delta)\right) \tag{21}$$

For small phase errors δ the cosine function can be approximated to one, hence the phase and the corresponding errors are

$$\sigma'' = \arctan\left(\tan\left(\frac{3\delta}{2} + \varphi\right) \cdot (2\cos(\delta) - 1)\right) \tag{22}$$

$$\sigma'' \approx \frac{3\delta}{2} + \varphi \tag{23}$$

$$\sigma''_{err} = \sigma'' - \varphi = \frac{3\delta}{2}. \tag{24}$$

Here, the error is three times higher than the dual ADC configuration. As for the dual configuration, the resulting errors in the case of a single ADC for several velocities in dependence of the sample rate are plotted in Fig. 5.

Although the error is constant for the two cases, it is not possible to remove it without knowing the movement of the target. Nevertheless, if the resulting error is acceptable, the system costs can be reduced, for example using microcontroller integrated ADCs, which often have only one or two ADCs working in parallel.

V. Conclusion

Six-Port based high distance resolution radar systems are often addressing the professional low cost market. Because an ADC can be one of the most expensive parts in such a system, this publication addresses the influence of the ADC selection to the overall system performance, in particular, the limitation of distance resolution due to the discretization as well as the decrease in resolution at configurations with only one or two sequentially sampling ADCs. Using the shown dependencies and *MATLAB* based simulation results, a coarse estimation of the needed ADC performance for different use cases can be done.

References

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