Segmental Polynomial Approximation based Phase Error Correction for Precise Near Field Displacement Measurements using Six-Port Microwave Interferometers

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***Abstract*— Six-Port microwave interferometers are a low-cost as well as low-power type of radar sensor with a high phase accuracy, which can be used for precise displacement measurements. Near field effects strongly influence the signal characteristics of a reflection of the electromagnetic wave near the antenna, especially if the target is low reflective. In this paper a calibration procedure based on phase error correction by segmental polynomial approximation is proposed that utilizes these effects. After validating the functionality of the calibration algorithm and its improvement by comparison to a comparable state-of-the-art procedure, two further near field measurements are presented. A cardboard as well as a plastic plate are used as low reflecting targets to show the applicability of the proposed calibration procedure for diverse measurement scenarios.**

***Index Terms*— Calibration technique; displacement measurements; interferometers; signal processing; Six-Port.**

I. Introduction

Radar systems have been used for navigation and tracking in civil and military tasks since decades. The fields of application have enlarged over the years, and nowadays, such sensors are used for medical and automotive applications as well as for industrial displacement measurements. A low-cost and low-power type of continuous wave (CW) radar sensor is, e.g., a Six-Port microwave interferometer. Originally used for power measurements [1], within a radar system the completely passive Six-Port structure superimposes the received electromagnetic signal with phase shifted portions of the reference signal to enable precise displacement measurements [2].

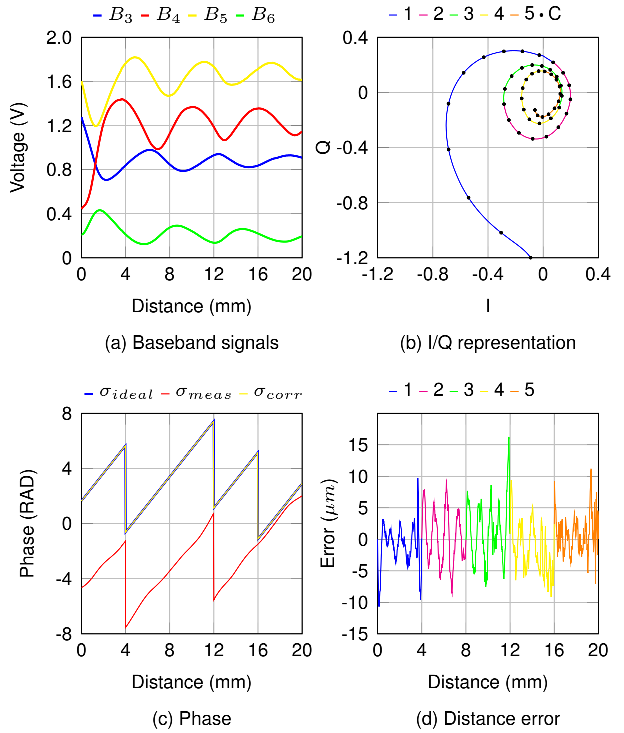
The power of the received signal strongly declines with increasing distance between antenna and target due to near field effects, which leads to a spiral shape of the complex representation of the measured baseband signals for displacement near the antenna [3], especially if the target is low reflective. Nonidealities and mismatches in the radio frequency (RF) part of the radar system further distort the baseband signals and thereby, also the complex representation. The resulting measurement error due to the subsequent incorrect phase evaluation can be minimized by prior calibration. Since hardware based approaches with known calibration standards, like open/short/match [4], imply a huge calibration effort, software based compensation methods only utilizing several measurements of the target at known positions are preferred [5]. In this paper a calibration procedure is proposed that piecewise corrects the distorted phase by segmental polynomial approximation. A big advantage of the presented algorithm is the very fast error correction due to the low mathematical effort during operation.

II. Calibration Procedure

The proposed calibration procedure is explained and validated with the measurement data published in [3] and compared with the state-of-the-art procedure of [5], a compensation method to optimize the reflection coefficient Γ, which also used that data for validation. Like [3], the proposed algorithm requires the complex representation $\underline{Z}$ of the baseband signals $B_{3\ldots6}$ shown in Fig. 1a:

$$\underline{Z} = I + jQ = (B_5 - B_6) + j(B_3 - B_4) \qquad (1)$$

Since $\underline{Z}$ is a contraction-free spiral, as depicted in the in-phase and quadrature (I/Q) plot in Fig. 1b, a piecewise compensation along the calibration range is possible. Similar to [5], several measurement points of known, ideally uniformly distributed target positions are required per calibration sub-range, at which the last calibration point of a sub-range equals the first one of the adjacent sub-range. In contrast to both other procedures, the proposed procedure does not directly operate on the baseband signals or the obtained I/Q data, but calculates the corresponding phase values σ_{meas} by arctangent demodulation. The resulting phase curves per sub-range are first unwrapped to guarantee monotonicity, and the ideal phase σ_{ideal} is calculated for each calibration point. In the main part of the calibration procedure the distorted phase curve shall be approximated with a polynomial function, whereby its inverse can be utilized for phase correction in subsequent measurements. Since the inverse of a polynomial is



(a) Baseband signals (b) I/Q representation

(c) Phase (d) Distance error

Fig. 1. This figure depicts (a) the measured baseband signals and (b) the offset corrected I/Q representation with dyed sub-ranges. The black measurement points C in (b) are used as calibration points for the proposed algorithm. The ideal phase as well as the measured and corrected phase are illustrated in (c). The remaining distance error after segmental phase correction is shown in (d).

numerically difficult to calculate, swapping the abscissa and ordinate values before approximation is far easier. Therefore, the least-squares curve fitting problem of the form

$$\min_{x} \frac{1}{2} \|C \cdot x - d\|_2^2 \tag{2}$$

is solved, where C represents the Vandermonde matrix for σ_{meas} and d equals the vector of σ_{ideal}. The number of columns of the Vandermonde matrix is determined by the desired order of the polynomial function plus one and additionally defines the minimum number of calibration points. Since the obtained polynomial tends to deflect between the boundary points and their corresponding neighbor, the closest calibration point inside of the adjacent sub-range(s) is also utilized for polynomial approximation. Thus, a total of 13 calibration points are used for the inner sub-ranges, and twelve for the two sub-ranges at the border of the measurement range. Since the phase is unique for only half the wavelength, a calibration sub-range has to be smaller than 6.25 mm for a 24 GHz radar system, as published in [3]. Here, the size of the sub-range is defined as 5.0 mm with eleven calibration points per sub-range, including both boundary points. The resulting sub-ranges for an approximation with a polynomial order of nine as well as

TABLE I
COMPARISON OF MAXIMUM (MAX) AND PEAK-TO-PEAK (P2P) ERRORS FOR DIFFERENT POLYNOMIAL ORDERS (PO).

PO	10	**9**	8	7	6	5	4	3
MAX (µm)	21	**16**	18	26	32	55	83	159
P2P (µm)	38	**27**	34	51	63	106	161	281

the calibration points are depicted in Fig. 1b, whereas the phase correction is shown in subplot (c). The remaining distance error is plotted in Fig. 1d along a measurement range of 20 mm and a step size of 20 µm. In comparison to [5], with a maximum error of 33 µm within a range of 3.125 ... 18.250 mm, the maximum error after phase error correction is less than 16 µm along the complete range. A comparison of the influence of the polynomial order on the maximum and peak-to-peak error in Tab. I shows that in this application an order of six is sufficient for an improvement. Utilizing Horner's method enables multiply-and-accumulate steps for polynomial evaluation, by what the polynomial order equals the number of computational steps, which is extremely fast and efficient. The significant decrease of computational effort is a major advantage of the proposed algorithm and enables an efficient realization in e.g., microcontrollers, or digital signal processors.

III. Near Field Measurements

A 24 GHz Six-Port microwave interferometer with a 4-by-4 patch array as antenna is utilized as monostatic radar sensor for further near field measurements. The four Six-Port output signals are down-converted by Schottky diodes, the resulting baseband voltages are sampled with four analog-to-digital converters and sent to a computer software, which controls a linear stage for displacement measurements and processes the acquired data. Sensor system and measurement setup are similar to [6], though, the radar sensor is attached to the linear stage and the target has a fixed position. Instead of a common aluminum target, a cardboard (CB) and a plastic plate (PP) are used as low reflecting targets in two measurement setups to create spiral shaped complex representations for displacements near the antenna. The measured baseband signals over a displacement range of 0 ... 70 mm and the corresponding offset corrected I/Q plots are depicted in Fig. 2 for both setups. As expected, the complex representation for measurements near the antenna has a deformed spiral shape. However, contractions appear only for distances larger than 35 mm, where the shape changes to an ellipse.

Similar to the calibration in Sec. II, a sub-range size of 5.0 mm is used for both measurement setups, which results in 14 independent calibrations to cover the complete measurement range of 70 mm. Eleven uniformly distributed calibration points per sub-range and a polynomial order of seven are used as a low quality (LQ) calibration setup, and 26 calibration points combined with a polynomial order of

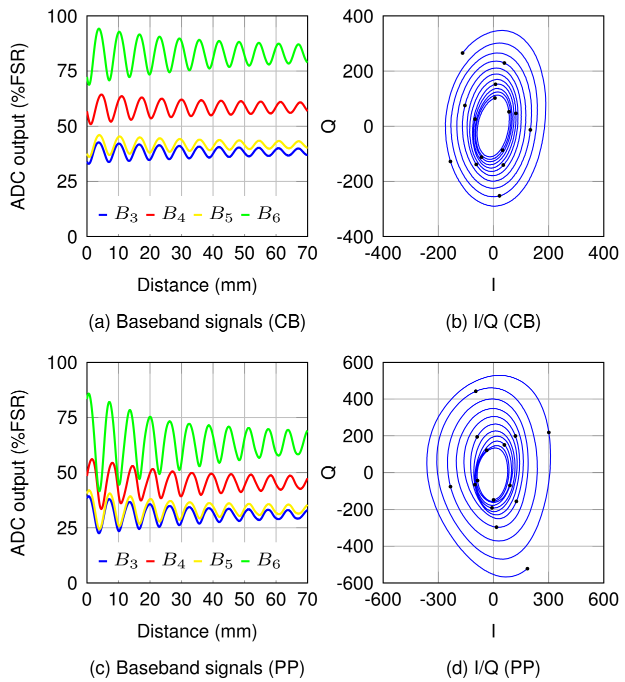


Fig. 2. Baseband signals and their offset corrected I/Q representation for both measurement setups.

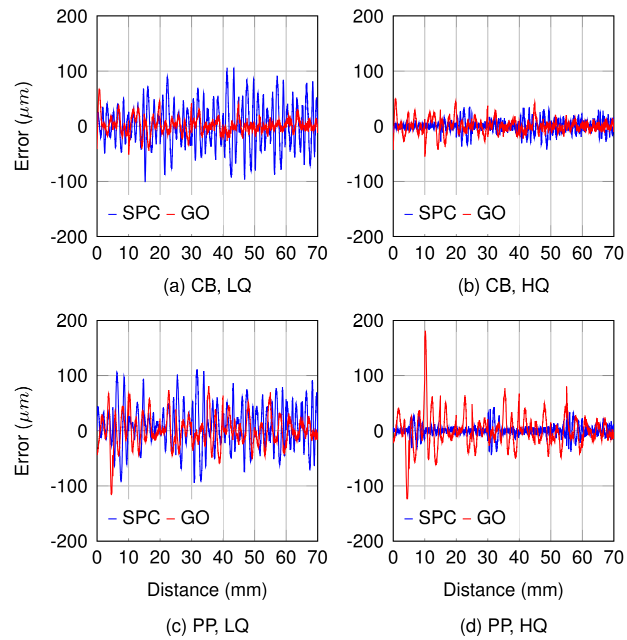


Fig. 3. Distance error comparison among SPC and GO for both measurement setups each with two different calibration qualities.

14 are used as a high quality (HQ) calibration setup. The remaining error after segmental phase correction (SPC) for both measurement setups and both calibration qualities is compared to the results of the gamma optimization (GO) of [5] in Fig. 3.

Especially in the near field, where a piece-wise calibration is feasible, the proposed procedure mostly obtains better results. Only for less deformed signal shapes and using few calibration points (Fig. 3a), the remaining error is smaller after GO. An increase of calibration points only slightly improves the performance of GO, but significantly enhances the performance of SPC by enabling a higher polynomial order, as shown in Tab. II. Regarding the computational effort, the evaluation of measurement data in *MATLAB* using GO requires roughly 40 s, whereas the SPC finishes in less than 1 ms.

TABLE II

COMPARISON OF MAXIMUM ERRORS FOR DIFFERENT MEASUREMENT AND CALIBRATION SETUPS.

Measurement target	CB		PP	
Calibration quality	LQ	HQ	LQ	HQ
Error after GO (µm)	**67**	55	116	181
Error after SPC (µm)	106	**42**	**112**	**44**

IV. CONCLUSION

In this paper a calibration procedure for precise near field measurements using Six-Port microwave interferometers was presented. The proposed algorithm minimizes the distance error by segmental polynomial approximation of the measured phase values. The functionality of the calibration procedure was validated by comparison to a comparable state-of-the-art procedure, at which the proposed algorithm could more than halve the distance error by contemporaneously decreasing the computational effort. The near field displace measurements with a cardboard as well as a plastic plate as target were presented to show the wide-ranging applicability of the algorithm.

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